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Inventors and Imposters

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firms into R&D**



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Inventors and impostors: an analysis of patent examination with self-selection of firms into R&D*

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Abstract

I present a model in which firms differing in R&D productivity choose between ambitious research projects, which are socially desirable, and unambitious ones, which are socially undesirable. The patent office must decide how rigorously to examine applications, which affects the probability of weeding out bad applications but also how firms self-select into more or less ambitious projects. Both the ex post and ex ante welfare effects need to be taken into account in determining the optimal examination intensity. The model allows me to assess the impact of various policy changes on examination and welfare, including the creation of specialized patent courts, post-grant opposition, and the delegation of fee-setting authority to the patent office. It generates a number of predictions that are consistent with empirical evidence on the patent system.

Keywords: innovation, patent office, optimal patent policy

JEL classification numbers: O31, O38, D73, D82, L50

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1 Introduction

Patent law requires that patents only be granted for inventions that are novel and nonobvious, and for good reason. Granting patents for inventions that are not novel causes deadweight loss and litigation without providing any offsetting benefit to society. Granting patents for inventions that do not satisfy a minimum inventive step may dampen the incentives of initial innovators or otherwise slow down the rate of technical progress (Scotchmer and Green, 1990; Scotchmer, 1996; O'Donoghue, 1998; Hunt, 2004). The patent office plays the role of a watchdog, making sure that only novel and nonobvious inventions obtain patent protection. Yet, as infringement lawsuits filed by holders of dubious patents against prominent firms such as eBay and RIM have brought to public awareness, the patent office does not reliably weed out bad patents.¹ The failure of the patent office to rigorously examine patent applications is a source of concern to many observers.² Lemley (2001), however, argues that the patent office may rationally choose to spend limited resources on examining a given application because only a tiny fraction of patents ever turn out to be commercially significant. The cost of weeding out more bad applicants might well exceed the benefit.

How rigorously should the patent office examine patent applications? What are the benefits and costs of more rigorous examination? Answering these questions requires a theory of the process through which applications are generated. In this paper, I propose a model in which firms choose between more or less ambitious research projects. Ambitious projects lead to social gains, while, if patented, unambitious projects lead to social losses. To obtain patent protection, firms have to file an application with the patent office. The patent office maximizes welfare and wields two instruments: an application fee and the intensity with which it examines applications; the combination of the two is what I define as *patent policy*. I first study how patent policy affects investment in R&D and characterize the optimal policy. I then use the model to assess the impact of various policy changes affecting the patent system on examination and welfare.

In the model presented in Section 2, firms differ in their ability to produce valuable inventions (their research productivity) and choose whether to start ambitious *research projects* or unambitious *bad projects*. Research projects may or may not lead to patentable inventions and are, in expectation, socially desirable. Bad projects never lead to patentable inventions and are socially undesirable. If applications on unpatentable inventions are accepted by the

¹ For example, RIM (Research In Motion), the maker of BlackBerry mobile devices, was sued by patent-holding company NTP, and settled out of court for a reported \$612.5 million, even though on re-examination the US Patent and Trademark Office (USPTO) revoked all of the patents NTP had asserted against RIM. See Time Magazine, "Patently Absurd", April 2, 2006, available online at <http://www.time.com/time/magazine/article/0,9171,1179349,00.html>.

² See, e.g., Jaffe and Lerner (2004), Farrell and Shapiro (2008) and Bessen and Meurer (2008).

patent office, they cause social losses. The private profitability of the two activities depends on firms' productivity and on the patent office's examination intensity, whose cost is increasing and convex. More rigorous examination makes it less likely that unpatentable inventions escape detection and therefore increases the attractiveness of research projects relative to bad projects. This setup leads to self-selection of firms: under a single-crossing condition, high-productivity firms choose research projects, while low-productivity firms choose bad projects or stay inactive. Importantly, the threshold above which firms choose research decreases with the examination intensity. My formulation thus acknowledges that the patent office may have a role in encouraging R&D, as stressed by Jaffe and Lerner (2004).

I show in Section 3 that the optimal patent policy involves full deterrence of bad projects. The examination intensity pins down the threshold for research; the application fee is set in such a way that all firms whose productivity is below the threshold remain inactive. To determine how rigorously to examine applications, the patent office equalizes the marginal benefit of patent examination with its marginal cost. The benefit consists of two parts: an *ex post* welfare effect due to fewer bad applications being granted and an *ex ante* effect whereby more rigorous examination increases the number of firms selecting into research.

If the patent office does not take into account the *ex ante* effect, it chooses a lower than optimal examination intensity. Even with a benevolent patent office, such behavior may arise because of a time inconsistency problem. I argue that the government may in that case want to retain fee-setting authority; by delegating it to the patent office, the government deprives itself of a tool to address the patent office's time inconsistency.

I derive comparative statics results concerning the optimal patent policy and use them to relate the creation of a specialized patent court to a decrease in examination intensity. Such a relationship is in line with the observation that the creation of the Court of Appeals for the Federal Circuit (CAFC) in the U.S. seems to have been accompanied by a decline in the rigor of examination on the part of the USPTO. I also investigate the effect of post-grant opposition and obtain a condition for an opposition procedure to be welfare-enhancing.

In Section 4, I extend the model by introducing wealth constraints on the part of some firms. I first consider the case where firms do not have access to credit. Wealth constraints then put an upper bound on the application fee that the patent office can charge. Provided the patent office is able to identify which firms are wealth constrained, it can condition its patent policy on the applicant's wealth. I show that the patent office will examine applications from wealth constrained firms more rigorously than those from other firms. Interpreting wealth constraints as being correlated with firm size, I derive predictions on differences in grant rates and patent values between small and large entities. These predictions are consistent with empirical evidence.

I go on to consider the case where firms have access to external funding. If preceded by a financing stage à la Holmstrom and Tirole (1997), the project-selection aspect of the basic model naturally gives rise to a moral-hazard problem. An entrepreneur who obtains a loan to finance a research project may be tempted to choose a bad project, which has a lower probability of success but is associated with larger private benefits. For the entrepreneur to have the incentive to choose the research project, he needs to be given a large enough share of the profit, thus limiting the size of the loan the firm can obtain. This creates an additional role for patent policy: more rigorous examination relaxes the entrepreneur's incentive-compatibility constraint, enabling more firms to get funding for R&D. I show that financing problems make it more costly to implement a given innovation threshold. Finally, Section 5 concludes.

Related literature

The paper contributes to the literature on optimal patent policy (see, e.g., Gilbert and Shapiro, 1990; Denicolò, 1996; Cornelli and Schankerman, 1999; Scotchmer, 1999; Hopenhayn and Mitchell, 2001; Hopenhayn *et al.*, 2006). Information asymmetries play a central role in this line of research, which builds on the observation that innovators are typically better informed about some dimensions of their innovation than the government. The literature has so far only been concerned with the cost and value dimension of innovation but ignored the novelty and non-obviousness dimension that is the focus of the present paper.

A number of recent papers investigate patent examination. Langinier and Marcoul (2009) study inventors' incentives to search for and disclose relevant prior art to the patent office. They find that, when the patent office cannot commit to a level of screening, there exists no equilibrium where applicants having obtained a positive signal separate from applicants with a negative signal in terms of the amount of prior art they submit. This is because the patent office has no incentive to search if it can identify valid applications beforehand. Caillaud and Duchêne (2011) present a model in which valid inventions stem from successful R&D projects and invalid ones from failed projects. They focus on the "overload problem" facing the patent office: when flooded with large numbers of applications, the average quality of examination declines, leading to a vicious circle by encouraging even more invalid applications. Again, there cannot be a separating equilibrium, i.e., one where only valid applicants file for a patent. Caillaud and Duchêne (2011) focus on the positive part of the analysis, leaving normative considerations about optimal examination intensity and application fees to the side.

Régibeau and Rockett (2010) examine the optimal duration of patent examination as a function of the importance of an innovation. They find that, controlling for the position in the innovation cycle, more important innovations should be examined faster, a prediction

which is consistent with evidence from a sample of U.S. patents. In a working paper version (Régibeau and Rockett, 2007), they consider the possibility that firms choose between genuinely innovative projects and non-inventions, as I do in this paper. However, they assume homogeneous applicants and focus on how to choose the delay of patent examination to provide incentives for firms to invest in genuine innovations, while I look at the optimal combination of examination effort and application fees.

Schuett (2010) examines the agency problem within the patent office. Patent examination is modeled as a moral-hazard problem followed by an adverse-selection problem: the examiner's incentives have to be structured so as to make the examiner exert effort searching for evidence to reject (prior art), but also to make him truthfully reveal whatever evidence (or lack thereof) he finds. It is argued that the model can explain the compensation scheme in use at the USPTO, where examiners are essentially rewarded for granting, as well as variation in compensation schemes and patent quality across patent offices.

Chiou (2008) looks at how the patent office's examination effort interacts with the incentives of private parties to bring court challenges. He shows that the two types of enforcement may be complementary, and that weak patents are more likely to be settled out of court than strong patents. His results call into question Lemley's (2001) rational ignorance argument.

2 The model

There is a continuum (with mass 1) of potential applicant firms. Each firm is characterized by a productivity parameter θ , which is its private knowledge and distributed according to a cumulative distribution function F with probability density f on the support $[0, \bar{\theta}]$. Assume that F is twice continuously differentiable and that $f(\theta) > 0$ for $\theta \in (0, \bar{\theta})$.

Firms choose between two activities, denoted R and B . Alternatively, they can remain inactive (denoted I). Activity R corresponds to a *research project* that can benefit society, while activity B corresponds to a *bad project* that, if patented, hurts society. Several interpretations are possible. One interpretation is that activity R is an ambitious R&D project that has the potential to lead to significant technical advances, while activity B is a project that only leads to minor improvements which do not justify the social cost of patent protection. Another interpretation is that activity R consists in genuine research, while activity B consists in submitting applications on existing technologies (or obvious combinations of existing technologies) without doing any R&D. Obtaining a patent on these existing technologies allows the patent holder to extract rents from users but does not benefit society.³

³ In practice, the possibility of challenging a patent in court mitigates this problem, but does not eliminate it if the court decision is uncertain. There may also be too little challenging of questionable patents because of the public good nature of these challenges (Chiou, 2006; Farrell and Shapiro, 2008).

Applications resulting from activity R are not easily distinguishable from applications resulting from activity B . The patent office needs to examine an application in order to obtain a signal about whether the claimed invention constitutes a significant advance over the prior art. The precision of the signal is determined by the patent office's *examination intensity* $e \in [0, 1]$. The cost of examining an application with intensity e is $\gamma(e)$, with $\gamma(0) = \gamma'(0) = 0$, $\gamma'(e) > 0$ for all $e > 0$, $\gamma'' > 0$, and $\gamma'(1) = \infty$.

2.1 General profit and welfare functions

Each firm chooses its activity from $\{R, B, I\}$. If a firm of type θ chooses R , its expected payoff given the patent office's examination intensity e is $\pi^R(\theta, e)$. If it chooses B , its payoff is $\pi^B(\theta, e)$. A firm's payoff from inactivity (I) is zero. The following assumption specifies how the profit functions are related to θ and e . Subscripts denote partial derivatives.

Assumption 1. *The profit functions satisfy the following conditions:*

- (i) $\pi_\theta^R > \pi_\theta^B \geq 0$, with strict inequality for $e < 1$,
- (ii) $\pi^R(0, 0) < \pi^B(0, 0)$ and $\pi^B(0, e) \geq 0$,
- (iii) $\pi^R(\bar{\theta}, 0) > \pi^B(\bar{\theta}, 0)$,
- (iv) $\pi_e^B < \pi_e^R \leq 0$.

Profits from both activities increase with θ , perhaps because both ambitious and not so ambitious research projects require some of the same qualities to be brought to fruition. Profits from activity R are more sensitive to productivity than those from activity B , though. When $e = 0$, it is more profitable for firms at the lower end of the productivity distribution ($\theta = 0$) to choose activity B , while for firms at the upper end of the distribution ($\theta = \bar{\theta}$), it is more profitable to choose R . Higher examination intensity negatively affects the profits from both activities, as the patent office is more likely to find defeating prior art; however, profits from activity B decrease faster with e than profits from R .

The social value accruing from a firm of type θ choosing R is $w(\theta, e)$, while the social value from the same firm choosing B is $-L(\theta, e)$. I assume $w_\theta \geq 0$, $w_e \geq 0$, $w_{ee} \leq 0$, and $L_e \leq 0$. That is, more productive firms create more valuable inventions; in addition, higher examination intensity increases the expected welfare from a given firm choosing R (at a decreasing rate) and reduces the expected social loss from choosing B . I also assume that the social value of activity R weakly exceeds its private value, $w(\theta, e) \geq \pi^R(\theta, e)$, and that activity B is socially harmful, $L(\theta, e) \geq 0$, with strict inequality for $e < 1$.⁴

⁴ The assumption that $w(\theta, e) \geq \pi^R(\theta, e)$ is not crucial but simplifies the exposition.

2.2 A specific example

To fix ideas, consider the following specification of profit and welfare functions, which will be analyzed in more detail below. Assume that activity R requires an investment k and leads to a major invention with probability ν and to a minor one with probability $1 - \nu$. At the time of applying for a patent, the firm does not know whether its research has generated a major or minor invention. Activity B requires no investment but always leads to minor inventions. If patented, the private value of a major invention is $g\theta$ and the private value of a minor invention is $b\theta$, with $g > b > 0$. The parameters g and b might capture, for example, the probability that a patent is challenged and overturned after being granted. More generally, g measures the profitability of major inventions, while b measures the ability of the holder of a bad patent to extract rents from the users of the patented technology. The social value of a major invention is $S > k$, while the social value of a minor invention is $-D < 0$.

The patent office receives a signal that depends on whether the application pertains to a major or a minor invention. Specifically, if an invention is minor, the patent office finds evidence (prior art) showing obviousness with probability e , and no such evidence with probability $1 - e$. If an invention is major, the patent office can never find evidence of obviousness. Consequently, the patent office rejects an application on a minor invention with probability e and never rejects applications on major inventions. With this specification, the profit functions are

$$\pi^R = \theta(\nu g + (1 - \nu)(1 - e)b) - k \quad (1)$$

$$\pi^B = (1 - e)b\theta. \quad (2)$$

The expected social welfare from activity R is

$$w = \nu S - (1 - \nu)(1 - e)D - k. \quad (3)$$

The expected social welfare from activity B is

$$-L = -(1 - e)D. \quad (4)$$

2.3 The effect of patent policy on firm behavior

The patent office maximizes welfare. Apart from the examination intensity e , the only other instrument at its disposal is an application fee ϕ that must be paid by all firms applying for patent protection. Given a patent policy (e, ϕ) , each firm chooses the activity that maximizes its expected payoff. Firm θ prefers activity R to B if

$$\pi^R(\theta, e) - \phi \geq \pi^B(\theta, e) - \phi.$$

Assumption 1 is sufficient for the existence of a unique threshold $\hat{\theta}$, defined by

$$\pi^R(\hat{\theta}, e) = \pi^B(\hat{\theta}, e), \quad (5)$$

such that, in the absence of application fees, activity B is chosen for all $\theta < \hat{\theta}$ and R is chosen for all $\theta \geq \hat{\theta}$. The threshold depends on e , i.e., $\hat{\theta} = \hat{\theta}(e)$. Assumption 1 implies that

$$\frac{\partial \hat{\theta}}{\partial e} = -\frac{\pi_e^R - \pi_e^B}{\pi_\theta^R - \pi_\theta^B} < 0. \quad (6)$$

Thus, the threshold above which firms choose R decreases with e , meaning that higher examination intensity leads a larger set of firms to choose ambitious R&D projects. The intuition is that stricter examination increases the relative attractiveness of ambitious research. Even though an increase in e reduces π^R , it reduces π^B even more, so that firms at the margin between B and R find it more profitable to choose R .

Provided $\phi \leq \pi^R(\hat{\theta}(e), e)$, there is a second threshold $\underline{\theta}$, defined by

$$\pi^B(\underline{\theta}, e) = \phi, \quad (7)$$

such that firms with $\theta \geq \underline{\theta}$ prefer B to inactivity. This threshold depends on both e and ϕ , i.e., $\underline{\theta} = \underline{\theta}(e, \phi)$. Firms with productivity below $\underline{\theta}$ remain idle, firms with productivity between $\underline{\theta}$ and $\hat{\theta}$ choose B , and firms with productivity above $\hat{\theta}$ choose R . By Assumption 1,

$$\frac{\partial \underline{\theta}}{\partial e} = -\frac{\pi_e^B}{\pi_\theta^B} > 0 \quad (8)$$

$$\partial \underline{\theta} / \partial \phi = 1 / \pi_\theta^B > 0. \quad (9)$$

Both instruments, e and ϕ , increase the threshold above which firms prefer activity B to inactivity. Higher examination intensity and higher application fees lead a smaller set of firms to choose bad projects. In summary, a patent policy (e, ϕ) leads to self-selection of firms between more or less ambitious R&D projects and inactivity, as illustrated in Figure 1.

3 Optimal patent policy

In this section, I first derive some general features of the optimal patent policy. I then derive further results under the specific functional forms set out in Subsection 2.2 above. The patent office chooses a patent policy (e, ϕ) , consisting of an application fee and an examination intensity, to maximize welfare, given by

$$\int_{\hat{\theta}(e)}^{\bar{\theta}} w(\theta, e) dF(\theta) - \int_{\underline{\theta}(e, \phi)}^{\hat{\theta}(e)} L(\theta, e) dF(\theta) - \gamma(e)[1 - F(\underline{\theta}(e, \phi))] \quad (10)$$

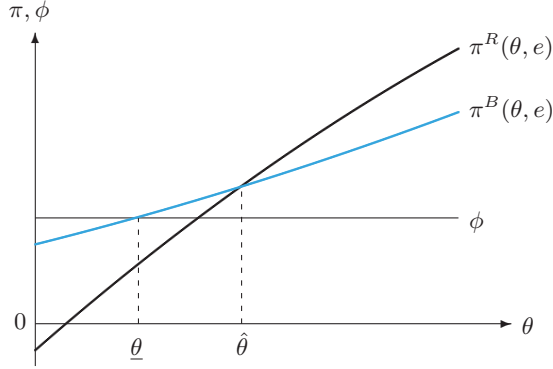


Figure 1: Self-selection of firms according to productivity θ

subject to $\underline{\theta}(e, \phi) \leq \hat{\theta}(e)$. The first term corresponds to the social value created by activity R (undertaken by firms whose productivity exceeds $\hat{\theta}$), the second term captures the expected social losses from bad patents, and the third term represents the cost of examination. The constraint $\underline{\theta} \leq \hat{\theta}$ reflects the fact that setting e and ϕ such that $\hat{\theta}$ is strictly below $\underline{\theta}$ can never be optimal. Holding ϕ constant, one could reduce e (and save the associated costs) without changing the set of firms that obtain patents. Moreover, the assumption that social value exceeds private value ensures that $w(\theta, e) \geq 0$ for all $\theta \geq \hat{\theta}$. The following proposition characterizes the optimal patent policy.

Proposition 1. *Suppose Assumption 1 holds. The optimal policy (e^*, ϕ^*) involves full deterrence of bad projects: $\underline{\theta} = \hat{\theta}$. The optimal examination intensity e^* is strictly positive and satisfies*

$$-\frac{\partial \hat{\theta}}{\partial e} w(\hat{\theta}, e^*) f(\hat{\theta}) + \int_{\hat{\theta}}^{\bar{\theta}} w_e(\theta, e^*) dF(\theta) = \gamma'(e^*) [1 - F(\hat{\theta})] - \frac{\partial \hat{\theta}}{\partial e} \gamma(e^*) f(\hat{\theta}). \quad (11)$$

The optimal application fee is $\phi^* = \pi^B(\hat{\theta}(e^*), e^*) > 0$.

Proof. Let us first show that the constraint $\underline{\theta} \leq \hat{\theta}$ must be binding. Let μ be the multiplier associated with the constraint. Differentiating (10) with respect to ϕ , we have

$$\frac{\partial \underline{\theta}}{\partial \phi} [f(\underline{\theta}) [L(\underline{\theta}, e) + \gamma(e)] - \mu] = 0. \quad (12)$$

Since $\partial \underline{\theta} / \partial \phi > 0$, $\mu > 0$, so indeed $\underline{\theta} = \hat{\theta}$. This implies $\phi = \pi^B(\hat{\theta}(e), e)$, which is strictly positive. We obtain (11) by differentiating (10) with respect to e , substituting for μ from (12) and using the fact that $\underline{\theta} = \hat{\theta}$.

What remains to be shown is that e^* is indeed interior. Evaluating the left-hand side of (11) at $e = 0$ yields

$$-\frac{\partial \hat{\theta}}{\partial e} w(\hat{\theta}(0), 0) f(\hat{\theta}(0)) + \int_{\hat{\theta}(0)}^{\bar{\theta}} w_e(\theta, 0) dF(\theta), \quad (13)$$

while the right-hand side is zero because $\gamma(0) = \gamma'(0) = 0$. Assumption 1 implies $\hat{\theta}(0) > 0$ and thus $f(\hat{\theta}(0)) > 0$. We know from (6) that $\partial \hat{\theta} / \partial e < 0$. By the definition of $\hat{\theta}$, $\pi^R(\hat{\theta}(0), 0) = \pi^B(\hat{\theta}(0), 0) > 0$. Hence, $w(\hat{\theta}(0), 0) \geq \pi^R(\hat{\theta}(0), 0) > 0$. Moreover, by assumption $w_e \geq 0$. Therefore, expression (13) is strictly positive, implying that welfare is strictly increasing in e at 0 and thus $e^* > 0$.

When evaluated at $e = 1$, the left-hand side of (11) is finite while the right-hand side tends to infinity because $\gamma'(1) = \infty$, implying that welfare is strictly decreasing in e at 1 and thus $e^* < 1$. As e^* is interior and welfare is continuous, (11) is a necessary condition for optimality. \square

The patent office chooses e^* to equalize the marginal social benefit of examination with its marginal cost and sets ϕ^* so as to deter all firms with $\theta < \hat{\theta}(e^*)$ from applying. At the optimum, only firms that given (e^*, ϕ^*) find it worthwhile to choose activity R apply for patents. All other firms remain inactive; no firm chooses activity B . Intuitively, raising the application fee up to $\phi = \pi^R(\hat{\theta}, e)$ does not represent a disincentive to innovation in this model: only those types of firms that would anyway find it optimal to choose bad projects are discouraged from applying for patents by such a fee. Thus, there is no loss in raising the fee up to the level where bad projects are completely deterred.

The marginal social benefit of examination (left-hand side of (11)) has two components. The first is the effect of stricter examination on the incentive to innovate ($-\frac{\partial \hat{\theta}}{\partial e} w(\hat{\theta}, e^*) f(\hat{\theta})$), as a larger e encourages more firms to choose R . The second is the effect of examination on the welfare generated by firms choosing R ($\int w_e(\theta, e) dF(\theta)$). Stricter examination enables the patent office to weed out more applications that do not warrant patent protection. The marginal cost of examination (the right-hand side of (11)) similarly consists of a direct effect of e on γ and an indirect effect on the number of applications (by reducing $\hat{\theta}$, an increase in e pushes up the number of applications).

Judging from casual observation, it seems that in practice firms sometimes do file applications which are not the outcome of extensive R&D or which they know not to satisfy the criteria for patentability. Take the famous example of the patent on the “peanut butter and jelly sandwich”⁵ (Jaffe and Lerner, 2004), issued to jam and jelly maker J.M. Smucker Co. It is hard to believe that this patent was the result of an ambitious research project. Not only

⁵ United States Patent No. 6,004,596, for a “sealed crustless sandwich.”

should it not have been issued, but the model suggests that, had the application fee been appropriately set, it should not have been applied for in the first place.

Two possible conclusions can be drawn from the existence of bad projects in practice. The first is that application fees are set too low and should thus be raised. One should be cautious not to rush to this conclusion, however, as the model is necessarily based on some simplifications. An alternative conclusion is that the model neglects certain relevant aspects of reality. One such aspect may be that innovative small firms, which are often short on cash, are unable to afford high application fees even though their expected future profits exceed those fees. I return to this issue in Section 4 below.

Proposition 1 assumes that the patent office maximizes welfare from an ex ante perspective. Firms' choice of activity, however, occurs before the patent office chooses e . Once applications are in, the examination intensity no longer affects $\hat{\theta}$. This may lead to a time inconsistency problem. The following proposition shows that when the patent office maximizes ex post welfare and neglects the effect of e on incentives to innovate, it tends to be too lax.

Proposition 2. *Suppose $w_{\theta e} \leq 0$. If the patent office does not take into account the effect of e on $\hat{\theta}$, it will choose a lower than optimal examination intensity $e < e^*$.*

Proof. When the patent office ignores the effect of e on $\hat{\theta}$, it chooses e to solve

$$\int_{\hat{\theta}}^{\bar{\theta}} w_e(\theta, e) dF(\theta) = \gamma'(e)[1 - F(\hat{\theta})]. \quad (14)$$

Assuming the patent office correctly anticipates the cutoff $\hat{\theta}$ chosen by applicants and sets $\phi = \pi^B(\hat{\theta}, e)$, the difference between (11) and (14) is

$$-\frac{\partial \hat{\theta}}{\partial e} f(\hat{\theta})[w(\hat{\theta}, e) - \gamma(e)].$$

Since $\partial \hat{\theta} / \partial e < 0$ by (6), it suffices to show that $w(\hat{\theta}(e^*), e^*) - \gamma(e^*) > 0$ to establish the claimed result. The first-order condition (11) implies that $\text{sign}(w - \gamma) = \text{sign}(\gamma'(1 - F) - \int w_e dF)$. I now show that this is incompatible with $w - \gamma \leq 0$ when $w_{\theta e} \leq 0$. Define e_0 as the value of e solving $w(\hat{\theta}, e) = \gamma(e)$. Thus,

$$\int_0^{e_0} [w_e(\hat{\theta}, e) - \gamma(e)] de = -w(\hat{\theta}, 0).$$

Because by assumption $w(\hat{\theta}, 0) \geq \pi^R(\hat{\theta}, 0) \geq 0$, $\gamma(0) = 0$ and $w_{ee} - \gamma'' < 0$, $w(\hat{\theta}, e) - \gamma(e) > 0$ if and only if $e < e_0$. Moreover, it must be the case that $e_0 > e_1$ defined by $w_e(\hat{\theta}, e_1) - \gamma'(e_1) = 0$.

We have

$$\int_{\hat{\theta}}^{\bar{\theta}} w_e(\theta, e) dF(\theta) = w_e(\hat{\theta}, e)(1 - F(\hat{\theta})) + \underbrace{\int_{\hat{\theta}}^{\bar{\theta}} (w_e(\theta, e) - w_e(\hat{\theta}, e)) dF(\theta)}_{\leq 0 \text{ because } w_{\theta e} \leq 0},$$

implying that $e_1 > e_2$ defined by $\int_{\hat{\theta}}^{\bar{\theta}} w_e(\theta, e_2) dF(\theta) / (1 - F(\hat{\theta})) - \gamma'(e_2) = 0$, and $\int_{\hat{\theta}}^{\bar{\theta}} w_e(\theta, e) dF(\theta) - \gamma'(e)(1 - F(\hat{\theta})) < 0$ if and only if $e > e_2$. Thus, $w - \gamma$ and $\gamma'(1 - F) - \int w_e dF$ being of the same sign requires $e_2 < e < e_0$. But this implies $w(\hat{\theta}, e^*) - \gamma(e^*) > 0$. \square

If the patent office behaves as an ex post welfare maximizer, it will examine applications less rigorously than ex ante welfare maximization would demand. This result relies on the patent office correctly anticipating the threshold $\hat{\theta}$ chosen by firms, and setting ϕ accordingly to deter firms below $\hat{\theta}$. It has implications for the question whether the patent office should have fee-setting authority. With the 2011 patent reform act, the U.S. Congress authorized the USPTO to set its own fees. The result in Proposition 2 suggests that if the patent office suffers from time inconsistency, the government may want to retain the authority to set fees. By setting ϕ lower than the patent office would, the government may be able to induce the patent office to screen applications more rigorously, thus bringing e closer to the optimal level; see Proposition 6 below.

The first-order condition of the welfare maximization problem (11) may have several solutions. While e^* must be one of the solutions, (11) does not completely pin down e^* in general. For completeness, the next proposition provides sufficient conditions for (11) to have a unique solution.

Proposition 3. *Suppose the distribution of θ has a nondecreasing hazard rate and that the profit functions satisfy $\pi_{\theta\theta}^R \leq \pi_{\theta\theta}^B$, $\pi_{ee}^R \leq \pi_{ee}^B$, and $\pi_{\theta e}^R \geq \pi_{\theta e}^B$. Then, equation (11) uniquely characterizes e^* if either $w_e = 0$ or $w_{\theta e} \leq 0$.*

Proof. See the Appendix.

Note that the assumptions on the second derivatives of π^R and π^B are all satisfied for the example of Subsection 2.2. The assumption that $w_{\theta e} \leq 0$, used in Propositions 2 and 3, also holds.⁶ The assumption that $w_e = 0$ holds in the example if $\nu = 1$, i.e., if ambitious research always results in patentable inventions.

3.1 Comparative statics

The following proposition provides comparative statics for the specification of profit and welfare functions from Subsection 2.2.

Proposition 4. *Suppose profit and welfare functions are as described in equations (1)-(4) and that the distribution of θ has a nondecreasing hazard rate. Letting $\hat{\theta}^* \equiv \hat{\theta}(e^*)$, we have the following comparative statics results:*

⁶ In the example, $w_{\theta} = 0$ because S and D are independent of θ . More generally, S and D could be functions of θ . For $w_{\theta} \geq 0$ and $w_{\theta e} \leq 0$, it is then sufficient that $S'(\theta) \geq 0$ and $D'(\theta) \leq 0$.

- (i) $\partial e^*/\partial g < 0$ and $\partial \hat{\theta}^*/\partial g < 0$,
- (ii) $\partial e^*/\partial b > 0$ and $\partial \hat{\theta}^*/\partial g > 0$,
- (iii) $\partial \hat{\theta}^*/\partial k > 0$,
- (iv) $\partial e^*/\partial S > 0$, $\partial \hat{\theta}^*/\partial S < 0$, and $\partial \phi^*/\partial S < 0$.

Proof. See the Appendix.

Recall that g is a parameter that shifts the private returns to innovation without changing the social returns. Similarly, the parameter b shifts the payoff from a bad patent. According to Proposition 4, an increase in g leads the patent office to examine applications less rigorously ($\partial e^*/\partial g < 0$). Nevertheless, a higher g leads to more research as some firms switch from activity B to activity R ($\partial \hat{\theta}^*/\partial g < 0$). An increase in b should result in a tightening of examination by the patent office ($\partial e^*/\partial b > 0$), yet would lead to a decline in research ($\partial \hat{\theta}^*/\partial b > 0$).

The comparative statics for the parameters g and b have implications for the introduction of a specialized appeals court for patent disputes. In 1982, the U.S. Congress established the Court of Appeals for the Federal Circuit (CAFC), which replaced the decentralized circuit courts in hearing appeals of patent cases. The European Union is currently considering a similar measure. How does the establishment of a specialized patent court affect the optimal patent policy? This depends on the effect of specialization on g and b . Formally,

$$de^* = \frac{\partial e^*}{\partial g} dg + \frac{\partial e^*}{\partial b} db.$$

One possibility is that a specialized court is able to appoint judges with greater expertise in the field of specialization. If more expertise on the part of judges reduces mistakes, a specialized court should lead to an increase in g and a decrease in b (i.e., $dg > 0$ and $db < 0$). Proposition 4 then implies that $de^* < 0$. Thus, the introduction of the CAFC should have led the patent office to optimally decrease its examination intensity. Katznelson (2007) shows that the USPTO grant rate rose from 60 percent in the early 1980s to 76 percent in 1998. One possible interpretation of this increase in the grant rate is that it reflects a decline in the rigor of patent examination, as several observers have suggested (see, e.g., Jaffe and Lerner, 2004).⁷ This observation is consistent with the model.

Another possibility is that a specialized patent court leads to a pro-patent shift, raising both g and b (i.e., $dg > 0$ and $db > 0$), which some commentators claim was the effect of the creation of the CAFC (Morrison, 1990; Jaffe and Lerner, 2004). In that case, the implications

⁷ Katznelson (2007) argues that other factors could also account for the increased grant rate. He cites an increase in the number of claims per application and a reduction in the scope of patents.

for e^* are generally ambiguous. Suppose that dg is small relative to db (perhaps because g is already close to 1). Then, a pro-patent CAFC should have led the patent office to examine applications more rigorously, rather than less.

Proposition 4 also shows that an increase in the cost of research (k) is associated with a reduction in the amount of research induced by the optimal patent policy, while an increase in the social value of innovation (S) leads to more research, tighter examination, and lower application fees.

3.2 Post-grant opposition

We can also use the model to assess the effects of post-grant opposition, an administrative procedure allowing third parties to challenge patents after they are granted. The European Patent Office (EPO) has long had a well-functioning opposition procedure, whereas the USPTO equivalent (called re-examination) has been rarely used (Graham *et al.*, 2002). Strengthening the re-examination procedure was one of the most prominent measures in the U.S. patent reform act of 2011.

Post-grant opposition is likely to reduce the costs of challenging a patent. Private parties may be knowledgeable about prior art that is relevant to the assessment of the novelty and nonobviousness of a claimed invention. Making it easier for these parties to bring such prior art to the attention of the authorities is the primary goal of an opposition procedure. By reducing the costs of challenging patents, however, opposition may also facilitate challenges against genuine inventions. Indeed, critics have expressed concerns that opposition would lead to abusive challenges and could thus reduce the incentive to innovate.

This discussion suggests that we should expect an opposition procedure to reduce both g and b : both good and bad patents will be challenged (and revoked) more often. It may be reasonable to think, however, that an effective opposition procedure would decrease b by more than g . The following definition formalizes this idea.

Definition. *Implementing an effective system of post-grant opposition leads to changes in g and b satisfying $db < dg \leq 0$.*

The model of self-selection into R&D presented in this paper shows that what matters for the incentive to innovate is not the absolute return from R&D but rather the return from R&D relative to the return from bad projects. Therefore, post-grant opposition may well enhance the incentive to innovate even though it reduces the return from R&D. In the absence of patent examination ($e = 0$), it is clear that $\hat{\theta}(0) = k/(\nu(g - b))$ decreases following

the introduction of an effective opposition procedure, in the sense defined above:

$$d\hat{\theta}(0) = \frac{\partial \hat{\theta}(0)}{\partial g} dg + \frac{\partial \hat{\theta}(0)}{\partial b} db = -\frac{k}{\nu(g-b)}(dg - db) < 0.$$

Nevertheless, as the following proposition shows, in the presence of patent examination, $db < dg$ is not sufficient for post-grant opposition to be welfare-enhancing.

Proposition 5. *An effective system of post-grant opposition improves total welfare if and only if $(1 - e)db \leq dg$.*

Proof. Total welfare under an optimal patent policy is

$$W^* \equiv \max_e (w(e) - \gamma(e))[1 - F(\hat{\theta}(e))].$$

By the envelope theorem,

$$\begin{aligned} \frac{\partial W^*}{\partial g} &= -\frac{\partial \hat{\theta}}{\partial g}(w(e) - \gamma(e))f(\hat{\theta}) \\ \frac{\partial W^*}{\partial b} &= -\frac{\partial \hat{\theta}}{\partial b}(w(e) - \gamma(e))f(\hat{\theta}). \end{aligned}$$

Therefore,

$$\begin{aligned} dW^* &= \frac{\partial W^*}{\partial g} dg + \frac{\partial W^*}{\partial b} db = -(w(e) - \gamma(e))f(\hat{\theta}) \left(\frac{\partial \hat{\theta}}{\partial g} dg + \frac{\partial \hat{\theta}}{\partial b} db \right) \\ &= -(w(e) - \gamma(e))f(\hat{\theta}) \frac{k(db(1 - e) - dg)}{\nu(g - (1 - e)b)^2}. \end{aligned} \tag{15}$$

Hence, a necessary and sufficient condition for $dW^* \geq 0$ is $db(1 - e) \leq dg$. \square

Proposition 5 says that the decrease in g brought about by introduction of opposition must be sufficiently small relative to the decrease in b to ensure that opposition is beneficial for welfare. Rewriting the condition, the ratio of the changes must satisfy

$$\frac{|dg|}{|db|} \leq 1 - e.$$

When e is low, i.e., the patent office does a poor job weeding out bad applications, effective opposition is likely to be welfare-enhancing. Part of the intuition for this result comes from how the patent office adapts its policy following the introduction of post-grant opposition. We know from Proposition 4 that the patent office tends to cushion the effect of a reduction in g on the incentive to innovate by increasing e ; similarly, it reacts to a reduction in b by decreasing e to save on examination costs.

Proposition 5 also provides grounds to be cautious about post-grant opposition, however. When e is large, i.e., the patent office does a good job of eliminating bad applications, even

an effective opposition procedure may reduce welfare. The reason is that a reduction in b has a smaller impact on firms' incentives to innovate than g . This is easy to see from $\hat{\theta} = k/(\nu(g - (1 - e)b))$. In the denominator, b is multiplied by $1 - e$ because it only matters if the firm escapes detection by the patent office. By contrast, g matters independently of e . Therefore the reduction in g can outweigh the reduction in b , in which case the introduction of an opposition procedure results in lower innovation and welfare.

4 Wealth constraints

The optimal policy identified by Proposition 1 (deterrence of all bad projects) may require substantial application fees. In practice, while proposals to raise fees are often welcomed by large companies, they are usually opposed by small inventors. A potential issue with high application fees is that small firms facing wealth constraints would be unable to afford them and would therefore forego investment in R&D altogether. To account for this issue, I now extend the model by assuming that a fraction of firms have limited wealth. I start by considering the case where firms do not have access to external finance in Section 4.1. I then turn to the case where wealth-constrained firms can obtain financing from outside investors in Section 4.2.

4.1 No access to credit

I modify the basic model as follows. Firms initially have wealth (or assets) A , which will be interpreted as cash on hand that can be used to finance an investment. A firm's assets can be high or low, $A \in \{A_\ell, A_h\}$. I will refer to firms with wealth A_ℓ as small and to firms with wealth A_h as large. The share of small firms is $\alpha < 1$. Assume that θ and A are independently distributed and that $k < A_\ell < k + \phi^* < A_h$, where ϕ^* is the optimal fee in the absence of wealth constraints and k denotes the sunk cost of investing in R&D, as in the example of Section 2.2. Thus, if the application fee is equal to ϕ^* , small firms are unable to pay for both the fee and the R&D investment needed to start a research project.

Suppose however that the patent office observes A , so that it can condition application fee and examination intensity on the applicant's wealth. The idea is that the patent office can observe whether an applicant is a small or a large firm, and that the size of the firm is correlated with the value of its assets. The patent office now chooses a policy towards small firms (e_ℓ, ϕ_ℓ) and a policy towards large firms (e_h, ϕ_h) . Clearly, since large firms can afford ϕ^* , the optimal policy towards large firms is the same as absent wealth constraints: $e_h = e^*$ and $\phi_h = \phi^*$. If the patent office wants small firms to invest in research as well, however, it can charge at most a fee of $A_\ell - k < \phi^*$. Provided small firms' expected research output is

sufficiently valuable, the patent office will indeed set $\phi_\ell = A_\ell - k$ (there is no reason to set a lower fee) and choose e_ℓ to maximize

$$\int_{\hat{\theta}(e)}^{\bar{\theta}} w(\theta, e) dF(\theta) - \int_{\underline{\theta}(e, A_\ell - k)}^{\hat{\theta}(e)} L(\theta, e) dF(\theta) - \gamma(e)[1 - F(\underline{\theta}(e, A_\ell - k))] \quad (16)$$

subject to $e \leq \bar{e}$, where \bar{e} solves $\hat{\theta}(\bar{e}) = \underline{\theta}(\bar{e}, A_\ell - k)$. Let e^{**} denote the value of e maximizing (16). The following proposition compares the optimal policy towards small firms to the one towards large firms (the latter being identical to the optimal policy absent wealth constraints).⁸

Proposition 6. *Let $\Delta \equiv \phi^* + k - A_\ell$. For low values of Δ , the optimal policy towards small firms involves a greater examination intensity than the optimal policy towards large firms: $e^{**} > e^*$.*

Proof. A necessary condition for the claimed result is that $\bar{e} > e^*$ for $\Delta > 0$. This condition holds because, by Proposition 1, $\bar{e} = e^*$ for $\Delta = 0$ (we have $\hat{\theta}(e^*) = \underline{\theta}(e^*, \phi^*)$) and

$$\frac{\partial \bar{e}}{\partial \Delta} = \frac{\partial \underline{\theta} / \partial \phi}{\partial \underline{\theta} / \partial e - \partial \hat{\theta} / \partial e} > 0,$$

where the inequality follows from (6), (8) and (9).

Differentiating (16) with respect to e and rearranging yields

$$\begin{aligned} & -\frac{\partial \hat{\theta}}{\partial e} w(\hat{\theta}, e) f(\hat{\theta}) + \int_{\hat{\theta}}^{\bar{\theta}} w_e(\theta, e) dF(\theta) - \gamma'(e)[1 - F(\hat{\theta})] + \frac{\partial \hat{\theta}}{\partial e} \gamma(e) f(\hat{\theta}) - \int_{\underline{\theta}}^{\hat{\theta}} L_e(\theta, e) dF(\theta) \\ & - \left(\frac{\partial \hat{\theta}}{\partial e} (L(\hat{\theta}, e) + \gamma(e)) f(\hat{\theta}) - \frac{\partial \underline{\theta}}{\partial e} (L(\underline{\theta}, e) + \gamma(e)) f(\underline{\theta}) \right) - \gamma'(e)[F(\hat{\theta}) - F(\underline{\theta})]. \quad (17) \end{aligned}$$

Evaluating (17) at e^* and $\Delta = 0$ so that $\hat{\theta} = \underline{\theta}$, we obtain

$$\left(\frac{\partial \underline{\theta}}{\partial e} - \frac{\partial \hat{\theta}}{\partial e} \right) (L(\hat{\theta}, e^*) + \gamma(e^*)) f(\hat{\theta}) > 0.$$

Thus, welfare is increasing in e at e^* when $\Delta = 0$. By continuity, this must also be true for Δ small but strictly positive. It follows that e^{**} must be strictly greater than e^* when $\Delta > 0$ at least in the vicinity of zero. Since $\bar{e} > e^*$ for $\Delta > 0$, the result holds irrespective of whether e^{**} is an interior solution or a corner solution. \square

According to Proposition 6, when a fraction of firms is wealth constrained and the patent office can identify those firms, it will examine their applications more rigorously than those

⁸ Note that I am implicitly assuming that the welfare from the optimal policy is greater than the welfare from shutting down small firms completely (zero). This is sure to be the case as long as Δ is not too large.

of firms that are not wealth constrained: $e_\ell = e^{**} > e_h = e^*$. The intuition is that because the patent office is forced to charge a lower application fee to small firms, it knows that, for a given e , the pool of applications from small firms will be of lower quality ($\underline{\theta} < \hat{\theta}^*$) than the pool of applications from large firms. The patent office thus has an additional incentive to tighten examination.

Note that $\phi_\ell < \phi^*$ does not necessarily mean that there will be bad projects in equilibrium. The patent office may well increase its examination intensity up to \bar{e} , at which point bad projects are again fully deterred. Whether or not bad projects are deterred, the proposition allows us to compare the equilibrium grant rates for small and large firms. Using the specific model set out in Section 2.2, the grant rate (GR) is

$$\text{GR} = \frac{(1 - F(\hat{\theta}))[\nu + (1 - \nu)(1 - e)] + (F(\hat{\theta}) - F(\underline{\theta}))(1 - e)}{1 - F(\underline{\theta})}.$$

When $\underline{\theta} = \hat{\theta}$, this simplifies to $\nu + (1 - \nu)(1 - e)$, which is decreasing in e . Moreover, $1 - e < \nu + (1 - \nu)(1 - e)$ (the grant rate for bad projects is lower than for research projects). Thus, irrespective of whether there is full deterrence of bad projects, the model predicts that the grant rate should be lower for small firms than for large firms.

Frakes and Wasserman (2012) report empirical results for the USPTO that are line with this prediction. Applicants in the U.S. receive differential treatment based on whether they are classified as small or large entities. In particular, since 1982 small entities pay reduced application and renewal fees. Frakes and Wasserman obtained data on USPTO patent processing outcomes disaggregated by patent class and entity size through a Freedom of Information Act request. They find evidence of a differential grant rate depending on whether an application is filed by a small or large entity. They argue that their estimates support the notion that the USPTO grants patents at a significantly higher rate to applicants with large-entity status.

The model also generates predictions on the average private value of patents held by small and large firms. Let a (“allowance”) denote the patent office’s decision to accept an application. By Bayes’ rule, the probability that a patent resulting from a research project is valid (i.e., corresponds to a major invention) is $\nu/(\nu + (1 - \nu)(1 - e))$. Thus, the expected value of a granted patent (EVP) is

$$\text{EVP} = \Pr(B|a)E(b\theta|\underline{\theta} \leq \theta < \hat{\theta}) + \Pr(R|a) \left[\frac{\nu E(g\theta|\theta \geq \hat{\theta})}{\nu + (1 - \nu)(1 - e)} + \frac{(1 - \nu)(1 - e)E(b\theta|\theta \geq \hat{\theta})}{\nu + (1 - \nu)(1 - e)} \right],$$

where $\Pr(B|a)$ is the probability that a firm has engaged in activity B given that it has been

granted a patent, and $\Pr(R|a)$ is defined analogously. Noting that

$$\begin{aligned}\Pr(B|a) &= 1 - \Pr(R|a) = \frac{(F(\hat{\theta}) - F(\underline{\theta}))(1 - e)}{(1 - F(\hat{\theta}))[\nu + (1 - \nu)(1 - e)] + (F(\hat{\theta}) - F(\underline{\theta}))(1 - e)}, \\ E(\theta|\underline{\theta} \leq \theta < \hat{\theta}) &= \frac{\int_{\underline{\theta}}^{\hat{\theta}} \theta dF(\theta)}{F(\hat{\theta}) - F(\underline{\theta})}, \\ E(\theta|\theta \geq \hat{\theta}) &= \frac{\int_{\hat{\theta}}^{\bar{\theta}} \theta dF(\theta)}{1 - F(\hat{\theta})},\end{aligned}$$

we obtain

$$\text{EVP} = \frac{[\nu g + (1 - \nu)(1 - e)b] \int_{\hat{\theta}}^{\bar{\theta}} \theta dF(\theta) + (1 - e)b \int_{\underline{\theta}}^{\hat{\theta}} \theta dF(\theta)}{[\nu + (1 - \nu)(1 - e)](1 - F(\hat{\theta})) + (1 - e)(F(\hat{\theta}) - F(\underline{\theta}))}. \quad (18)$$

Given Proposition 6, do small firms hold more or less valuable patents than large firms on average? To answer this question, suppose that the optimal examination intensity towards small firms is a corner solution, $e^{**} = \bar{e}$, so that there are no bad projects. (Clearly, when there are bad projects, the average value of a patent is lower.) Then, (18) simplifies to

$$\text{EVP} = \frac{\int_{\hat{\theta}}^{\bar{\theta}} \theta dF(\theta)}{1 - F(\hat{\theta})} \frac{\nu g + (1 - \nu)(1 - e)b}{\nu + (1 - \nu)(1 - e)}.$$

The first term is the expected value of θ for firms doing research. It increases with $\hat{\theta}$ and hence decreases with e . The derivative of the second term with respect to e is $\nu(1 - \nu)(g - b)/(\nu + (1 - \nu)(1 - e))^2 \geq 0$; the sign reflects the fact that tightening examination increases the posterior probability that a granted patent is valid. A sufficient condition for small firms to hold on average less valuable patents than large firms thus is that ν is not too different from 1.

This result is consistent with the findings in Bessen (2008), who uses renewal data to estimate the value of a sample of U.S. patents, controlling for a number of patentee characteristics. Bessen reports that patents owned by patentees with small entity status are substantially less valuable than those owned by large entities. He argues that the difference remains sizeable even when accounting for the selection bias that is likely to be present because small firms tend to sell their most valuable patents. Similar results are obtained by Arora *et al.* (2008) using survey estimates.

4.2 Patent policy and firms' financing problem

An obvious objection to the analysis of wealth constraints in Section 4.1 is that in the presence of well-functioning financial markets, firms' wealth should not matter. We know that credit markets do not work perfectly, however, and this is particularly salient in the financing of

innovation. This section explicitly models the firms' financing problem. I build on the previous section by introducing a financing stage for firms whose assets are insufficient to fund the R&D investment. This allows me to study how patent policy affects firms' access to financing.

It is well known from the corporate finance literature that an entrepreneur needs to have sufficient pledgeable income to obtain a loan from outside investors (Holmstrom and Tirole, 1997). The reason that is generally invoked is moral hazard: after obtaining the loan, the entrepreneur may have to choose between projects that differ in their probability of success and in the private benefits they procure the entrepreneur. In order for the entrepreneur to choose the efficient project, rather than the one maximizing his private benefits, he has to be given a sufficient share of the profits.

The model of self-selection into R&D from Section 2 fits this description: firms – which we can interpret as being run by entrepreneurs in this context – must choose between two kinds of projects, one of which (B) has a lower probability of success (because it is more likely to be rejected by the patent office or revoked by the courts) but requires a lower investment and may thus be associated with larger private benefits. In what follows, I will use the terms “firm” and “entrepreneur” interchangeably.

In the basic model, firms with productivity close to $\hat{\theta}$ make very little profit, as $\phi^* = \pi^R(\hat{\theta}, e^*)$. As the preceding discussion suggests, in the presence of a financing problem, these firms may not be able to secure a loan. To induce them to invest in research, the patent office must leave them with sufficient “rent” by setting $\phi < \pi_R(\hat{\theta})$. I will now formalize this idea.

Assume that wealth constrained firms have assets A_ℓ satisfying $0 \leq A_\ell < k$. Thus, in order to finance the R&D investment required for activity R , firms need outside financing. By contrast, activity B requires outside financing only if the application fee exceeds agents' assets, $\phi > A_\ell$. In order to make a profit from either kind of project, firms need to obtain a patent. The filing of an application to the patent office is publicly observable.

There is a competitive financial market where investors can provide funding to firms. The investors have an alternative investment (outside option) whose rate of return is normalized to zero. The outcome of a project is uncertain: a research project succeeds with probability p , while a bad project succeeds with probability pb , where $0 < b \leq 1$. In case of success, the project of entrepreneur θ – be it a research project (R) or a bad project (B) – yields θ/p . In case of failure, the project yields zero.⁹ For simplicity, I assume that investors observe a firm's productivity θ but not its choice of project.¹⁰

If an entrepreneur obtains a loan of size σ but does not spend its entirety on R&D and

⁹ Compared to the specific model from Section 2.2 analyzed in previous sections, here I have normalized g to 1 and assume for simplicity that research always yields patentable inventions ($\nu = 1$).

¹⁰ This assumption is made for reasons of tractability; it avoids complications arising from signaling.

application fees, he can divert the remainder to other uses, which for simplicity is assumed to procure him a private benefit equal to the amount of money that is left over. This private benefit is only realized if the investors do not shut down the project before its outcome is realized.

The timing is as follows:

1. The entrepreneur makes a take-it-or-leave-it offer to investors, specifying a loan of size σ and a repayment in case of success ρ . Investors accept or reject.
2. The entrepreneur chooses R (in which case he invests k) or B . He then applies for a patent and pays the application fee ϕ . The patent office examines applications with intensity e and grants or rejects. Research projects are never rejected while bad projects are rejected with probability $1 - e$. If an application is rejected the entrepreneur obtains zero.
3. The project succeeds or fails. In case of success, the entrepreneur pays back ρ .

I solve the model backwards starting from the moral-hazard stage.

Stage 2: choice of activity. An entrepreneur who has obtained a loan of size $\sigma < k + \phi - A_\ell$ cannot choose R and therefore decides between B and I . B is more profitable if and only if

$$(1 - e)pb(\theta/p - \rho) - \phi + \sigma \geq 0. \quad (19)$$

Because patent applications are observable, the investors can shut down the project if the firm does not apply for a patent, so the right-hand side is zero.

An entrepreneur who has obtained a loan of size $\sigma \geq k + \phi - A_\ell$ chooses among R , B , and I . He prefers R to B if and only if

$$p(\theta/p - \rho) - k - \phi + \sigma \geq (1 - e)pb(\theta/p - \rho) - \phi + \sigma. \quad (20)$$

While a research project R has a higher probability of success, it also requires a larger investment (k). Expression (20) can be rewritten as

$$\rho \leq \frac{\theta}{p} - \frac{k}{p(1 - (1 - e)b)}. \quad (21)$$

The entrepreneur prefers R to I if and only if $p(\theta/p - \rho) - k - \phi + \sigma \geq 0$ (investors shut down the project, depriving the entrepreneur of his private benefit, if he does not apply for a patent).

Stage 1: loan proposal. The entrepreneur will propose the loan contract (σ, ρ) that allows investors to just break even. An entrepreneur who anticipates choosing R will propose a contract such that $\sigma = k + \phi - A_\ell = p\rho$. Investors only accept this contract if it indeed induces the entrepreneur to choose R . If the contract were to induce B , the probability of repayment would be too low for investors to break even. This places an upper bound on ρ . Replacing $\rho = (k + \phi - A_\ell)/p$ in (20), we obtain

$$k + \phi - A_\ell \leq \theta - \frac{k}{1 - (1 - e)b} \Leftrightarrow \theta \geq \frac{k(2 - (1 - e)b)}{1 - (1 - e)b} + \phi - A_\ell \equiv \tilde{\theta}. \quad (22)$$

If this condition is not met, even the lowest possible reimbursement (such that investors just break even in expectation) cannot ensure that the entrepreneur chooses R . According to this definition, $\tilde{\theta}$ is the threshold above which firms have the ability to innovate, i.e., they can obtain financing to do research.

An entrepreneur who anticipates choosing B and requires a loan (i.e., if $A_\ell < \phi$) will propose $\sigma = \phi - A_\ell = (1 - e)p b \rho$. By proposing $\sigma < k + \phi - A_\ell$, the entrepreneur effectively commits to choosing B . Because patent applications are observable, there is no moral hazard, so all such loan contracts are accepted. If $A_\ell \geq \phi$, the entrepreneur does not require a loan. In both cases, (19) implies that it is profitable to choose B if and only if

$$(1 - e)b\theta - \phi \geq 0 \Leftrightarrow \theta \geq \underline{\theta} = \frac{\phi}{(1 - e)b}. \quad (23)$$

We are now ready to describe the effect of patent policy on the behavior of wealth-constrained firms. Given a patent policy (e, ϕ) , firm θ prefers activity R to activity B – provided it can obtain financing – if

$$p\theta/p - k - \phi \geq (1 - e)b p \theta/p - \phi \Leftrightarrow \theta \geq \hat{\theta} = \frac{k}{1 - (1 - e)b}.$$

The firm's moral-hazard problem, however, implies that it can only obtain financing if $\theta \geq \tilde{\theta}$. We have $\tilde{\theta} > \hat{\theta}$ if and only if

$$\frac{k(2 - (1 - e)b)}{1 - (1 - e)b} + \phi - A_\ell > \frac{k}{1 - (1 - e)b} \Leftrightarrow A_\ell < k.$$

Thus, if small firms' wealth is lower than the investment required for R&D, then regardless of the patent policy (e, ϕ) there exists a nonempty set of firms that would find it profitable to do research but cannot obtain financing.

The threshold $\tilde{\theta}$, above which firms have both the incentive and the ability to innovate, depends on e and ϕ as follows:

$$\begin{aligned} \frac{\partial \tilde{\theta}}{\partial e} &= -\frac{bk}{(1 - (1 - e)b)^2} < 0 \\ \frac{\partial \tilde{\theta}}{\partial \phi} &= 1 > 0. \end{aligned}$$

Unlike in the model without financing, the threshold above which firms innovate also depends on ϕ ; $\tilde{\theta} = \tilde{\theta}(e, \phi)$ decreases in e and increases in ϕ .

Patent policy thus has an effect on firms' ability to obtain funding for research. Raising the examination intensity improves a given firm's chance to get funding. The intuition is that more rigorous examination relaxes the entrepreneur's incentive-compatibility constraint (20): it decreases the success probability of activity B , making deviations less attractive for the entrepreneur. This increases the amount of income that can be pledged to investors and hence encourages investors to provide the entrepreneur with funds for research.

As previously, the threshold $\underline{\theta}$ above which firms prefer activity B to inactivity increases in both instruments, $\partial \underline{\theta} / \partial e = \phi / (b(1 - e)^2) > 0$ and $\partial \underline{\theta} / \partial \phi = 1 / ((1 - e)b) > 0$. Again, the model generates self-selection of firms according to their productivity. Firms with $\theta < \underline{\theta}$ remain inactive, firms with $\underline{\theta} \leq \theta < \tilde{\theta}$ choose activity B , and firms with $\theta \geq \tilde{\theta}$ choose activity R .

An implication of this analysis is that if the patent office implements the patent policy that is optimal in the basic model (where $\underline{\theta} = \hat{\theta}$), wealth-constrained entrepreneurs with $\theta \in [\hat{\theta}, \tilde{\theta})$ are credit rationed: they cannot obtain a loan covering $k + \phi$. As a result they are forced to choose bad projects. Although they would like to invest in research projects, they cannot obtain the required funding. By contrast, they can always obtain a loan for a bad project whenever it is profitable to do so. Since $\underline{\theta} = \hat{\theta}$, activity B is profitable for all entrepreneurs that are credit rationed. This suggests that the patent office has to adapt its patent policy to account for firms' financing problems either by lowering fees or raising its examination intensity, or both. The following proposition makes this claim precise.

Proposition 7. *In the presence of financing problems, achieving a given level of innovation $\tilde{\theta} \geq 2k - A_\ell$ requires a lower application fee and greater examination intensity than in the basic model. In choosing the application fee, the patent office trades off research projects against bad projects: $\partial(\tilde{\theta} - \underline{\theta}) / \partial \phi < 0$.*

Proof. Suppose the patent office wants to implement an innovation threshold of θ' . For this to be feasible with financing, θ' must be larger than $\tilde{\theta}$ evaluated at $\phi = 0$ and $e = 1$, i.e., $\theta' \geq 2k - A_\ell$. In the basic model, implementing θ' requires

$$\hat{\theta}(e) = \frac{k}{1 - (1 - e)b} \leq \theta' \Leftrightarrow e \geq 1 - \frac{\theta' - k}{b\theta'} \equiv \hat{e}$$

and $\phi \leq \theta' - k \equiv \hat{\phi}$. With financing, implementing θ' requires

$$\tilde{\theta}(e, 0) = \frac{k(2 - (1 - e)b)}{1 - (1 - e)b} - A_\ell \leq \theta' \Leftrightarrow e \geq 1 - \frac{\theta' - 2k + A_\ell}{b(\theta' - k + A_\ell)} \equiv \tilde{e}$$

and

$$\tilde{\theta}(1, \phi) = \frac{k(2-b)}{1-b} + \phi - A_\ell \leq \theta' \Leftrightarrow \phi \leq \theta' - k + A_\ell - \frac{k}{1-b} \equiv \tilde{\phi}.$$

Straightforward computations show that $\tilde{e} > \hat{e}$ if and only if $k(A_\ell - k) < 0$ and that $\tilde{\phi} < \hat{\phi}$ if and only if $A_\ell - k/(1-b) < 0$, both of which are implied by the assumption that $A_\ell < k$.

For the second claim, taking the derivative of

$$\tilde{\theta} - \hat{\theta} = \frac{k(2 - (1-e)b)}{1 - (1-e)b} + \phi - A_\ell - \frac{\phi}{(1-e)b}$$

with respect to e yields $\partial(\tilde{\theta} - \hat{\theta})/\partial e = -(1 - (1-e)b)/((1-e)b) < 0$, where the inequality follows from $b \leq 1$. \square

The proposition shows that when firms are wealth-constrained, inducing a given level of innovation is costlier than absent wealth constraints: the patent office has to set a lower fee and higher examination intensity. Unlike in the basic model, where raising the application fee up to $\pi^R(\hat{\theta}, e)$ has no effect on innovation, here the patent office faces a tradeoff. On the one hand, raising ϕ reduces the number of firms choosing bad projects (firms with $\theta \in [\underline{\theta}, \tilde{\theta})$). On the other hand, it pushes up the threshold above which firms choose research projects, thus diminishing innovation. To keep the level of innovation constant, an increase in ϕ must be accompanied by an increase in e .

Note that it is still possible, and may be desirable, for the patent office to fully deter bad projects. In the presence of financing problems, this requires choosing e and ϕ such that $\underline{\theta}(e, \phi) = \tilde{\theta}(e, \phi)$. Since $\tilde{\theta} > \hat{\theta}$ for any ϕ , achieving full deterrence necessitates a larger e and is thus costlier than in the basic model, however. Depending on parameters, it will sometimes be preferable to allow some bad projects and save on examination costs.

5 Conclusion

I have presented a model of patent examination in which firms self-select into R&D depending on their productivity. Specifically, firms choose between more or less ambitious research projects and then apply for patent protection. Ambitious research projects are socially desirable and often yield patentable inventions. By contrast, unambitious projects are socially harmful and never result in patentable inventions. The patent office, charged with the verification of patentability, must separate the wheat from the chaff. It has two instruments at its disposal: the intensity with which it examines applications, and an application fee. The self-selection feature of the model implies that higher examination intensity allows the patent office to weed out more bad applications (an ex post effect) but also affects the set of firms that choose ambitious research projects (an ex ante effect). Both the ex post and

ex ante welfare effects need to be taken into account in determining the optimal examination intensity.

I have shown that the optimal patent policy involves full deterrence of bad projects. The examination intensity is set so as to equalize the marginal social benefits from examination with the marginal social cost, and pins down the threshold above which firms choose ambitious research. The application fee is set to discourage all firms below the threshold from applying for patents. I have used the model to assess the impact of various policy changes on examination and welfare, including the creation of a specialized patent court, post-grant opposition, and delegation of fee-setting authority to the patent office. I have also derived a number of predictions which are consistent with empirical evidence on the patent system. Finally, I have extended the model by adding a financing stage. Firms' choice between projects then leads to a moral hazard problem that patent policy can help to solve.

Appendix

Proof of Proposition 3. What needs to be shown is that the second-order condition for a maximum holds at any e that solves (11), implying that there cannot be a local minimum, which in turn implies that there must be a unique maximum. Dropping the arguments of functions and integral bounds for brevity, the second-order condition is

$$-(w-\gamma) \left[\frac{d^2\hat{\theta}}{de^2} f + \left(\frac{\partial\hat{\theta}}{\partial e} \right)^2 f' \right] - \left(\frac{\partial\hat{\theta}}{\partial e} \right)^2 w_\theta f + \int w_{ee} dF - \gamma''(1-F) + 2(\gamma' - w_e) \frac{\partial\hat{\theta}}{\partial e} f < 0. \quad (24)$$

If e solves (11), then $-(\partial\hat{\theta}/\partial e)(w-\gamma) = (\gamma'(1-F) - \int w_e dF) / f$, which we can use to rewrite (24) as

$$\begin{aligned} - (w-\gamma) \frac{d^2\hat{\theta}}{de^2} f + \frac{\partial\hat{\theta}}{\partial e} \frac{2(\gamma' - w_e) f^2 + [\gamma'(1-F) - \int w_e dF] f'}{f} - \left(\frac{\partial\hat{\theta}}{\partial e} \right)^2 w_\theta f \\ + \int w_{ee} dF - \gamma''(1-F) < 0. \end{aligned} \quad (25)$$

We know from (6) that $\partial\hat{\theta}/\partial e < 0$. Moreover,

$$\frac{d^2\hat{\theta}}{de^2} = \frac{(\pi_e^R - \pi_e^B) \left[\frac{\partial\hat{\theta}}{\partial e} (\pi_{\theta\theta}^R - \pi_{\theta\theta}^B) + \pi_{\theta e}^R - \pi_{\theta e}^B \right] - (\pi_\theta^R - \pi_\theta^B) \left[\frac{\partial\hat{\theta}}{\partial e} (\pi_{\theta e}^R - \pi_{\theta e}^B) + \pi_{ee}^R - \pi_{ee}^B \right]}{(\pi_\theta^R - \pi_\theta^B)^2} \geq 0,$$

where the inequality follows from Assumption 1 and the assumption that $\pi_{\theta\theta}^R \leq \pi_{\theta\theta}^B$, $\pi_{ee}^R \leq \pi_{ee}^B$, and $\pi_{\theta e}^R \geq \pi_{\theta e}^B$.

Consider first the case $w_e = 0$. Expression (25) then simplifies to

$$-(w - \gamma) \frac{d^2 \hat{\theta}}{de^2} f + \frac{\partial \hat{\theta}}{\partial e} \gamma' \frac{2f^2 + (1 - F)f'}{f} - \left(\frac{\partial \hat{\theta}}{\partial e} \right)^2 w_\theta f - \gamma''(1 - F) < 0.$$

The last two terms are negative by assumption. Moreover, if $w_e = 0$, then the first-order condition (11) implies that $w - \gamma > 0$, so the first term is negative as well. The assumption on the hazard rate means that

$$\frac{d}{d\theta} \left(\frac{f}{1 - F} \right) = \frac{f^2 + (1 - F)f'}{(1 - F)^2} \geq 0,$$

implying that the second term is also negative.

Now consider the case $w_{\theta e} \leq 0$. We know from the proof of Proposition 2 that $w_{\theta e} \leq 0$ implies $w(\hat{\theta}(e^*), e^*) - \gamma(e^*) > 0$, $\int w_e dF \leq w_e(1 - F)$, and $\gamma' - w_e > 0$. Thus, a sufficient condition for (25) is

$$-(w - \gamma) \frac{d^2 \hat{\theta}}{de^2} f + \frac{\partial \hat{\theta}}{\partial e} (\gamma' - w_e) \frac{2f^2 + (1 - F)f'}{f} - \left(\frac{\partial \hat{\theta}}{\partial e} \right)^2 w_\theta f + \int w_{ee} dF - \gamma''(1 - F) < 0.$$

The last three terms are negative by assumption. Owing to the fact that $w - \gamma > 0$, the first term is also negative. Finally, combined with the fact that $\gamma' - w_e > 0$, a nondecreasing hazard rate again suffices for the second term to be negative. \square

Proof of Proposition 4. Consider general profit and welfare functions π^R , π^B , and w . Let $e(\hat{\theta})$ denote the implicit function defined by $\pi^R(\hat{\theta}, e) = \pi^B(\hat{\theta}, e)$ (i.e., $e(\hat{\theta})$ is the inverse function of $\hat{\theta}(e)$). Assume $w_\theta = 0$ and let

$$W(\theta, e) \equiv (w(e) - \gamma(e))[1 - F(\hat{\theta})].$$

Let m be a parameter affecting profits or welfare.

Lemma 1. *Suppose $\partial \hat{\theta} / \partial m \neq 0$ and $\text{sign}(\partial^2 \hat{\theta} / \partial m \partial e) = -\text{sign}(\partial \hat{\theta} / \partial m)$. If one of the following holds, then $\text{sign}(\partial e^* / \partial m) = \text{sign}(\partial \hat{\theta} / \partial m)$:*

- (a) $w_m = 0$,
- (b) $w_{em} = 0$ and $\text{sign}(w_m) = \text{sign}(\partial \hat{\theta} / \partial m)$,
- (c) $\text{sign}(w_m) = \text{sign}(w_{em}) = \text{sign}(\partial \hat{\theta} / \partial m)$.

Suppose instead $\partial \hat{\theta} / \partial m = 0$. If $w_{em} = 0$ or $\text{sign}(w_{em}) = \text{sign}(w_m)$, then $\text{sign}(de^ / dm) = \text{sign}(w_m)$.*

Proof. Let $\tilde{W} \equiv W(\hat{\theta}(e), e)$. By Proposition 1, $e^* = \arg \max_e \tilde{W}$ is an interior solution, implying that at e^* , $\partial \tilde{W} / \partial e = 0$ and $\partial^2 \tilde{W} / \partial e^2 < 0$. Thus,

$$\frac{de^*}{dm} = -\frac{\partial^2 \tilde{W} / \partial e \partial m}{\partial^2 \tilde{W} / \partial e^2}$$

has the sign of $\partial^2 \tilde{W} / \partial e \partial m$. We have

$$\begin{aligned} \frac{\partial^2 \tilde{W}}{\partial e \partial m} &= \frac{\partial^2 \hat{\theta}}{\partial e \partial m} \frac{\partial W}{\partial \hat{\theta}} + \frac{\partial \hat{\theta}}{\partial m} \left[\frac{\partial \hat{\theta}}{\partial e} \frac{\partial^2 W}{\partial \hat{\theta}^2} + \frac{\partial^2 W}{\partial \hat{\theta} \partial e} \right] + \frac{\partial \hat{\theta}}{\partial e} \frac{\partial^2 W}{\partial \hat{\theta} \partial m} + \frac{\partial^2 W}{\partial e \partial m} \\ &= -\frac{\partial^2 \hat{\theta}}{\partial e \partial m} (w - \gamma) f + \frac{\partial \hat{\theta}}{\partial m} \left[-\frac{\partial \hat{\theta}}{\partial e} (w - \gamma) f' + (\gamma' - w_e) f \right] - \frac{\partial \hat{\theta}}{\partial e} w_m f + (1 - F) w_{em} \\ &= -\frac{\partial^2 \hat{\theta}}{\partial e \partial m} (w - \gamma) f + \frac{\partial \hat{\theta}}{\partial m} (\gamma' - w_e) \frac{(1 - F) f' + f^2}{f} - \frac{\partial \hat{\theta}}{\partial e} w_m f + (1 - F) w_{em}. \end{aligned}$$

By the assumption on the hazard rate, the fraction is nonnegative. We know from the proof of Proposition 3 that $w_\theta = 0$ implies that $w - \gamma$ and $\gamma' - w_e$ are positive. Recalling that $\partial \hat{\theta} / \partial e < 0$, the result is immediate. \square

Lemma 2. Suppose $\partial e / \partial m \neq 0$ and either $\partial^2 e / \partial m \partial \hat{\theta} = 0$ or $\text{sign}(\partial^2 e / \partial m \partial \hat{\theta}) = -\text{sign}(\partial e / \partial m)$. If one of the following holds, then $\text{sign}(\partial \hat{\theta}^* / \partial m) = \text{sign}(\partial e / \partial m)$:

- (a) $w_m = 0$,
- (b) $w_{em} = 0$ and $\text{sign}(w_m) = -\text{sign}(\partial e / \partial m)$,
- (c) $\text{sign}(w_m) = \text{sign}(w_{em}) = -\text{sign}(\partial e / \partial m)$.

Suppose instead $\partial e / \partial m = 0$. If $w_{em} = 0$ or $\text{sign}(w_{em}) = \text{sign}(w_m)$, then $\text{sign}(\partial \hat{\theta}^* / \partial m) = -\text{sign}(w_m)$.

Proof. Let $\hat{W} \equiv W(\hat{\theta}, e(\hat{\theta}))$. The optimal threshold $\hat{\theta}^*$ can be defined as $\hat{\theta}^* = \hat{\theta}(e^*)$ or, alternatively, as $\hat{\theta}^* = \arg \max_{\hat{\theta}} \hat{W}$, which must satisfy $\partial \hat{W} / \partial \hat{\theta} = 0$ and $\partial^2 \hat{W} / \partial \hat{\theta}^2 < 0$. Thus,

$$\frac{d\hat{\theta}^*}{dm} = -\frac{\partial^2 \hat{W} / \partial \hat{\theta} \partial m}{\partial^2 \hat{W} / \partial \hat{\theta}^2}$$

has the sign of $\partial^2 \hat{W} / \partial \hat{\theta} \partial m$. We have

$$\begin{aligned} \frac{\partial^2 \hat{W}}{\partial \hat{\theta} \partial m} &= \frac{\partial^2 e}{\partial \hat{\theta} \partial m} \frac{\partial W}{\partial e} + \frac{\partial e}{\partial m} \left[\frac{\partial e}{\partial \hat{\theta}} \frac{\partial^2 W}{\partial e^2} + \frac{\partial^2 W}{\partial \hat{\theta} \partial e} \right] + \frac{\partial^2 W}{\partial \hat{\theta} \partial m} + \frac{\partial e}{\partial \hat{\theta}} \frac{\partial^2 W}{\partial e \partial m} \\ &= -\frac{\partial^2 e}{\partial \hat{\theta} \partial m} (\gamma' - w_e)(1 - F) + \frac{\partial e}{\partial m} \left[\frac{\partial e}{\partial \hat{\theta}} (w_{ee} - \gamma'')(1 - F) + (\gamma' - w_e) f \right] \\ &\quad - w_m f + \frac{\partial e}{\partial \hat{\theta}} (1 - F) w_{em}. \end{aligned}$$

By the inverse function theorem, $\partial e / \partial \hat{\theta} = 1 / (\partial \hat{\theta} / \partial e) < 0$. We know from the proof of Proposition 3 that $w_\theta = 0$ implies $\gamma' - w_e > 0$. Moreover, $w_e e - \gamma'' < 0$ by assumption. Combining these yields the result. \square

I now apply Lemmata 1 and 2 to the specific functional forms set out in (1) through (4). Note that

$$\begin{aligned}\hat{\theta}(e) &= \frac{k}{\nu(g - (1 - e)b)} \\ e(\hat{\theta}) &= \frac{k}{\nu b \hat{\theta}} - \frac{g - b}{b}.\end{aligned}$$

Claim (i). We have $\partial \hat{\theta} / \partial g = -k / (\nu(g - (1 - e)b)^2) < 0$, $\partial^2 \hat{\theta} / \partial g \partial e = 2bk / (\nu(g - (1 - e)b)^3) > 0$, $\partial e / \partial g = -1/b < 0$, $\partial^2 e / \partial g \partial \hat{\theta} = 0$, and $w_g = 0$. By Lemma 1, $\partial e^* / \partial g < 0$. By Lemma 2, $\partial \hat{\theta}^* / \partial g < 0$.

Claim (ii). We have $\partial \hat{\theta} / \partial b = (1 - e)k / (\nu(g - (1 - e)b)^2) > 0$, $\partial^2 \hat{\theta} / \partial b \partial e = -k(g + (1 - e)b) / (\nu(g - (1 - e)b)^3) < 0$, $\partial e / \partial b = -(k - \nu g \hat{\theta}) / (\nu b^2 \hat{\theta}) > 0$, $\partial^2 e / \partial b \partial \hat{\theta} = k / (\nu b^2 \hat{\theta}^2) > 0$, and $w_b = 0$. By Lemma 1, $\partial e^* / \partial b > 0$. By Lemma 2, $\partial \hat{\theta}^* / \partial b > 0$.

Claim (iii). We have $\partial e / \partial k = 1 / (\nu b \hat{\theta}) > 0$, $\partial^2 e / \partial k \partial \hat{\theta} = -1 / (\nu b \hat{\theta}^2) < 0$, $w_k = -1$, and $w_{ek} = 0$. By Lemma 2, $\partial \hat{\theta}^* / \partial k > 0$.

Claim (iv). We have $\partial \hat{\theta} / \partial S = \partial e / \partial S = 0$, $w_S = v > 0$, and $w_{eS} = 0$. By Lemma 1, $\partial e^* / \partial S > 0$. By Lemma 2, $\partial \hat{\theta}^* / \partial S < 0$. Finally, recalling from Proposition 1 that $\phi^* = \pi_B(\hat{\theta}^*, e^*) = (1 - e^*)b\hat{\theta}^*$, the previous results imply

$$\frac{\partial \phi^*}{\partial S} = b \left[(1 - e^*) \frac{\partial \hat{\theta}^*}{\partial S} - \hat{\theta}^* \frac{\partial e^*}{\partial S} \right] < 0.$$

\square

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